Math 60 12.2 Circles

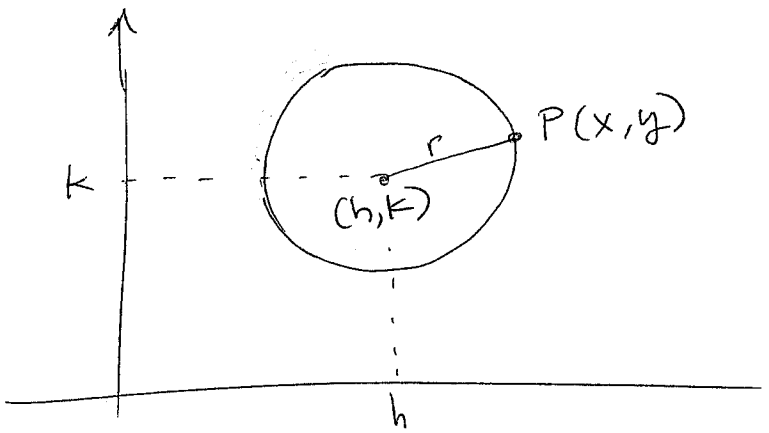
Objectives 1) Write the standard form for the equation of a circle.

2) Graph a circle

3) Find the center and radius of a circle given its equation in general form.

Definition

A circle is a set of all points that are equally distant from the center of the circle — a fixed point we'll call (h,k).



The distance from a point on the circle to the center is called the radius r.

If (h,k) is the center and (x,y) is a point on the circle and the distance between them is r, then the distance formula gives

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

Square both sides

* Memorize

$$(x-h)^2 + (y-k)^2 = r^2$$

standard form of equation of a circle with center (h,k) and radius r.

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① Write the standard form of the equation of a circle having center $(-2,5)$ and radius 3.

$(x-h)^2 + (y-k)^2 = r^2$ circle formula.

$(x-(-2))^2 + (y-5)^2 = 3^2$ substitute (h,k) = center = $(-2,5)$, $r=3$

$\boxed{(x+2)^2 + (y-5)^2 = 9}$ simplify inside ( ) only.

② Graph the circle $(x+3)^2 + (y-2)^2 = 16$

step 1: Identify the center.

caution The formula has subtract, so the coordinates are the opposites of the numbers you see.

$(x+3)^2 + (y-2)^2 = 16$

x coordinate next to x-variable. -3

y-coordinate next to y-variable. 2

Center $(-3,2)$

step 2: Identify the radius.

caution The formula has square, so take the square root of the RHS.

$(x+3)^2 + (y-2)^2 = 16$

$r = \sqrt{16} = 4$

$r = 4$

cont

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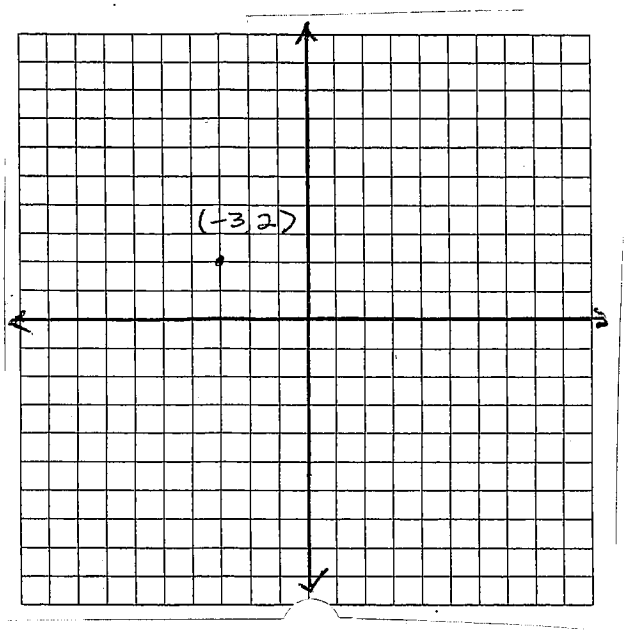
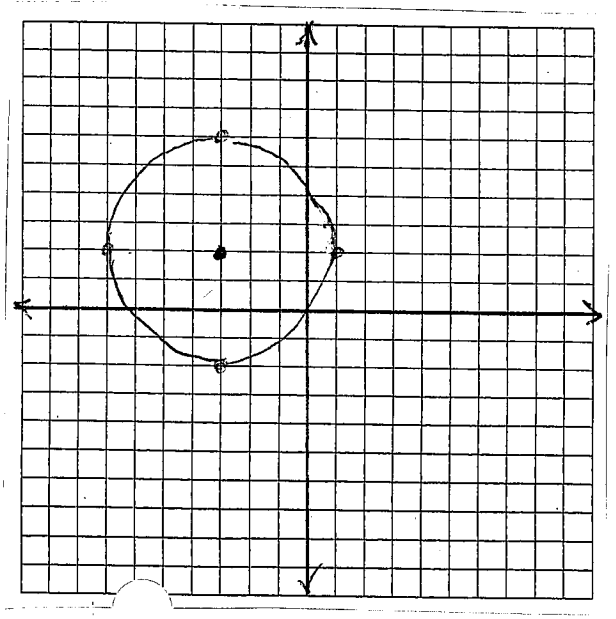
Step 3: Plot the center on graph paper.



Step 4: Use the radius to measure the correct distance to 4 points on the circle

up
down
left
right



Step 5: Free-hand a circle to make a neat, round connection.

(3) Find the center and radius of $x^2+y^2=144$

notice that $x^2=(x-0)^2$
and $y^2=(y-0)^2$

$(x-0)^2+(y-0)^2=12^2$

center (0,0)
radius 12

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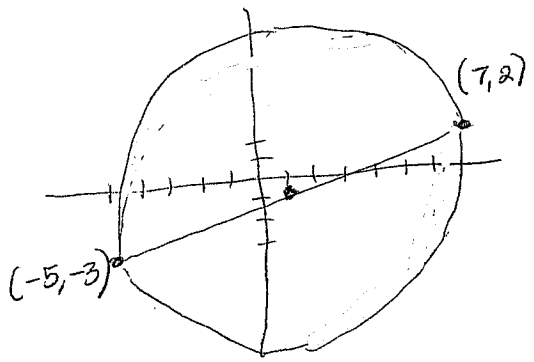
④ Find the center and radius of $(x-2)^2 + (y+2)^2 = \frac{1}{4}$

center $(2, -2)$

radius $= \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$ = radius

⑤ Find the equation of a circle in standard form having endpoints of a diameter at $(-5,-3)$ and $(7,2)$.



remember: A diameter must pass through the center of the circle.

The center is halfway between the endpoints of any diameter.
= center is midpoint.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\left(\frac{-5+7}{2}, \frac{-3+2}{2}\right)$$

$$\left(\frac{2}{2}, \frac{-1}{2}\right)$$

$$\left(1, -\frac{1}{2}\right) \text{ center}$$

The radius is the distance from the center to any point on the circle or half the length of a diameter.

option 1: center to point $(7,2)$

distance formula $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(7-1)^2 + \left(2-\frac{-1}{2}\right)^2}$

$= \sqrt{6^2 + \left(2+\frac{1}{2}\right)^2}$

$= \sqrt{6^2 + \left(\frac{5}{2}\right)^2}$

cont

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$$= \sqrt{36 + \frac{25}{4}}$$

$$= \sqrt{\frac{144+25}{4}}$$

$$= \sqrt{\frac{169}{4}}$$

$$\boxed{r = \frac{13}{2}}$$

option 2: center to $(-5, -3)$

$$r = \sqrt{(1-(-5))^2 + (-\tfrac{1}{2}-(-3))^2}$$

$$= \sqrt{(1+5)^2 + (-\tfrac{1}{2}+3)^2}$$

$$= \sqrt{6^2 + (\tfrac{5}{2})^2}$$

$$= \sqrt{36 + \frac{25}{4}} = \sqrt{\frac{169}{4}} = \frac{13}{2}$$

option 3: half of distance from $(-5, -3)$ to $(7, 2)$

$$r = \frac{1}{2}\sqrt{(7-(-5))^2 + (2-(-3))^2}$$

$$r = \frac{1}{2}\sqrt{(7+5)^2 + (2+3)^2}$$

$$r = \frac{1}{2}\sqrt{12^2 + 5^2}$$

$$r = \frac{1}{2} \cdot \sqrt{144+25}$$

$$r = \frac{1}{2} \cdot \sqrt{169}$$

$$r = \frac{1}{2} \cdot 13 = \frac{13}{2}$$

Equation of Circle $(x-h)^2 + (y-k)^2 = r^2$

$$(x-1)^2 + (y-(-\tfrac{1}{2}))^2 = (\tfrac{13}{2})^2$$

$$\boxed{(x-1)^2 + (y+\tfrac{1}{2})^2 = \frac{169}{4}}$$

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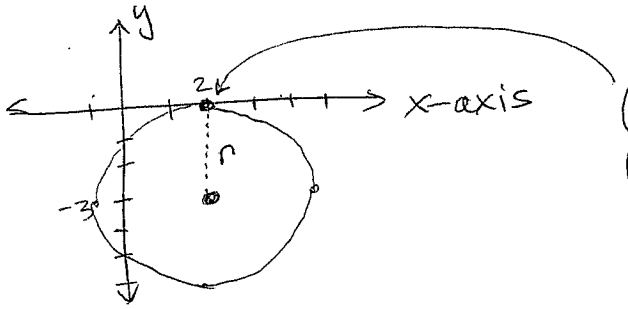
⑥ Find the standard form of the equation of a circle with center at $(2,-3)$ and tangent to the x-axis.

To understand the question, draw a graph.



circle should touch the x-axis here $(2,0)$.

distance from $(2,0)$ to $(2,-3)$ is $3.$ = radius

equation of circle $(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-(-3))^2=3^2$

$\boxed{(x-2)^2+(y+3)^2=9}$

⑦ Find the area and circumference of the circle $(x-1)^2+(y-4)^2=49.$

Area of circle $A=\pi r^2$ only requires radius!

$r=\sqrt{49}=7$

$A=\pi(7)^2$

$\boxed{A=49\pi \text{ sq. units}}$

Circumference of circle $C=2\pi r$ only requires radius!

$r=7$

$C=2\pi(7)$

$\boxed{C=14\pi \text{ units}}$

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Objective 3) Find the center and radius of a circle whose equation is given in general form.

Review:

#P (8) Find the center and radius of $x^2 + (y-3)^2 = 64$

$(x-0)^2$	$(y-3)^2$	r^2
x-coord	y-coord	radius
0	3	8

center (0, 3)
radius 8

(9) Simplify $(x+2)^2 + (y-3)^2 = 25$

A.K.A. Write the equation in <u>general form</u>.

$$ax^2 + by^2 + cx + dy + e = 0.$$

$(x+2)(x+2) + (y-3)(y-3) = 25$

$x^2+4x+4 + y^2-6y+9 = 25$

FOIL FOIL

$x^2+y^2+4x+6y+13-25=0$

$$\boxed{x^2+y^2+4x-6y-12=0}$$

Find the center and radius, then graph.

(10) $x^2+y^2-4x+6y-3=0$

Need to complete the square <u>twice</u>

- once for the x^2 and x
- once for the y^2 and y

<u>step 1</u>: Collect x^2 and x, y^2 and y, constant to RHS

$x^2-4x + y^2+6y = 3$

<u>step 2</u>: CTS for x, add to both sides

$\# = \frac{-4}{2} = -2$ $\quad x^2-4x+4 + y^2+6y = 3+4$

$\#^2 = (-2)^2 = 4$ $\quad (x-2)^2 + y^2+6y = 7$

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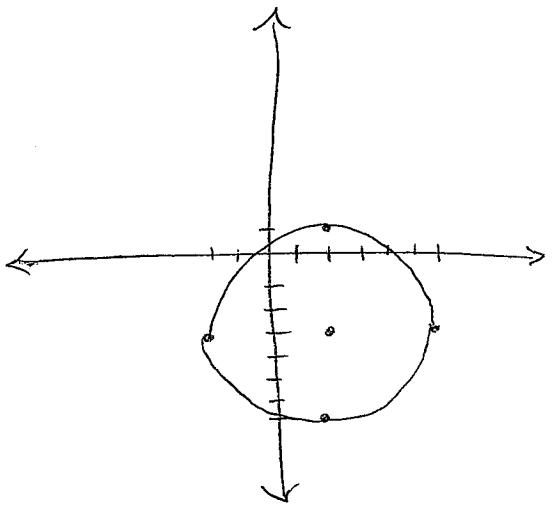
step 3: (May be combined with step 2)
CTS for y, add to both sides

$(x-2)^2 + y^2+6y+9 = 7+9$

$\# = \frac{6}{2} = 3$

$\#^2 = 3^2 = 9$

$(x-2)^2 + (y+3)^2 = 16.$



center $(2,-3)$
radius $\sqrt{16} =$ 4 radius

(11) $x^2+y^2+8x-2y+6=0$

collect $x^2+8x+16 + y^2-2y+1 = -6+16+1$

$\# = \frac{8}{2}=4$ $\quad$ $\# = \frac{-2}{2}=-1$

$\#^2 = 4^2=16$ $\quad$ $\#^2 = (-1)^2=1$

$(x+4)^2+(y-1)^2 = 11$

center $(-4,1)$
radius $\sqrt{11}$

(12) $2x^2+2y^2-12x-8y-24=0$

Recall: CTS DOES NOT WORK if NOT $1x^2$ or $1y^2$.

Divide entire equation by 2.

$\frac{2x^2}{2} + \frac{2y^2}{2} - \frac{12x}{2} - \frac{8y}{2} - \frac{24}{2} = \frac{0}{2}$

$x^2+y^2-6x-4y-12=0$

$x^2-6x+9 + y^2-4y+4 = 12+9+4$

$\# = \left(\frac{-6}{2}\right) = -3$ $\quad$ $\# = \left(\frac{-4}{2}\right) = -2$

$\#^2 = (-3)^2 = 9$ $\quad$ $\#^2 = (-2)^2 = 4$

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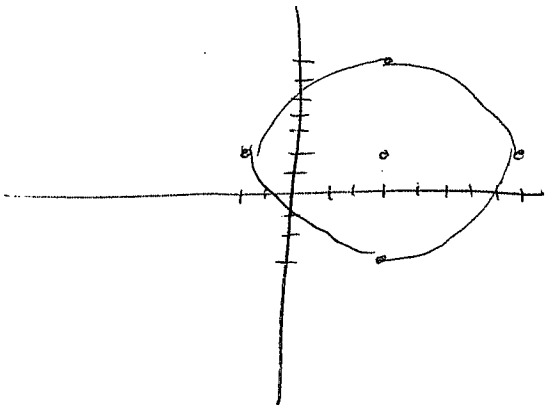
$(x-3)^2+(y-2)^2=25$

center $(3,2)$
radius 5



(13) $3x^2+3y^2+6x+24y+48=0$

Divide all by 3.

$$\frac{3x^2}{3}+\frac{3y^2}{3}+\frac{6x}{3}+\frac{24y}{3}+\frac{48}{3}=\frac{0}{3}$$

$$x^2+y^2+2x+8y+16=0$$

$$x^2+2x+1+y^2+8y+16=-16+1+16$$

$\#=\frac{2}{2}=1$ $\quad$ $\#=\frac{8}{2}=4$

$\#^2=1^2=1$ $\quad$ $\#^2=4^2=16$

$$(x+1)^2+(y+4)^2=1$$

center $(-1,-4)$
radius 1